EE 4343/5329 - Control System Design Project
LECTURE 6

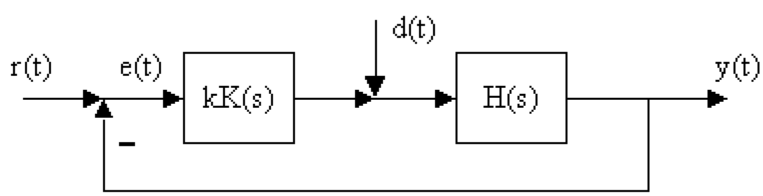
Updated: Tuesday, October 12, 1999

PID/LEAD/LAG COMPENSATORS

When the characteristics of a plant are not suitable, they can be changed by adding a compensator in the control system. Some basic sorts of compensators useful in feedback control design are discussed, including Proportional-Integral-Derivative (PID), lead, and lag.

Compensator Transfer Functions

A tracking control system is given in the figure. The tracker topology is often used since it has a unity-gain outer loop that often aides in analysis and understanding. The plant to be controlled is H(s) and the compensator is kK(s), with k a constant gain. The function of this controller is to make the output follow the reference input r(t) by keeping small the tracking error e(t)=r(t)-y(t). The disturbance is denoted as d(t).



The closed-loop transfer function is

$$T(s) = \frac{Y(s)}{R(s)} = \frac{kK(s)H(s)}{1+kK(s)H(s)}.$$

The denominator

$$\Delta(s) = 1 + kK(s)H(s) \equiv 1 + kG(s)$$

is a polynomial fraction whose numerator is the closed-loop characteristic polynomial. G(s) is the open-loop gain.

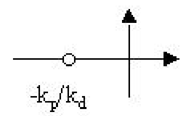
In guaranteeing stability and performance and shaping the closed-loop response, it is important to select a suitable compensator kK(s). Several common sorts of compensators are given below.

Proportional (P)

$$kK(s) = k, \quad const$$

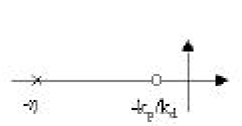
Proportional-plus-Derivative (PD)

$$kK(s) = k_d s + k_p = k_d (s + {}^{k_p}\!/_{k_d})$$



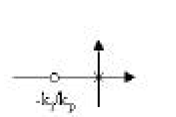
Practical Proportional-plus-Derivative (PD)

$$kK(s) = k_d \frac{s \quad {}^{k}\!/_{k_d}}{s+h}$$, where η is a large filtering pole



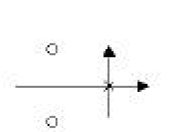
Proportional-plus-Integral (PI)

$$kK(s) = k_p + \frac{k_i}{s} = \frac{k_p s + k_i}{s} = k_p \frac{s + {}^{k_i}\!/_{k_p}}{s}$$



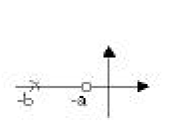
PID

$$kK(s) = k_d s + k_p + \frac{k_i}{s} = k_d \frac{s^2 + \frac{k_p}{k_d} s + \frac{k_i}{k_d}}{s}$$



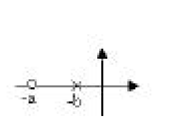
Lead

$$kK(s) = k \frac{s+a}{s+b}, \quad a < b$$



Lag

$$kK(s) = k \frac{s+a}{s+b}, \quad a > b$$



Example- Double Integrator With Different Compensators

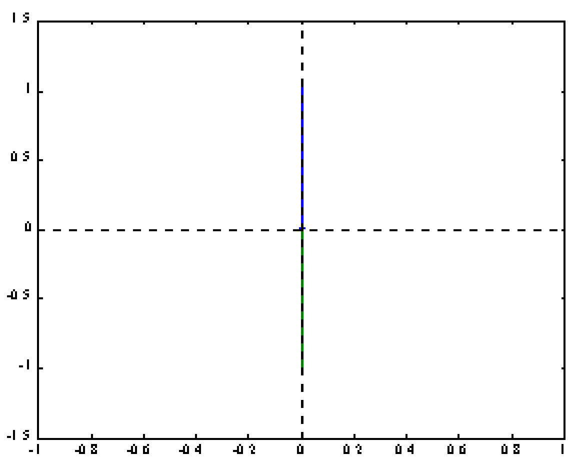
Suppose the plant is given by the double integrator H(s)= $1/s^2$. This is representative of many mechanical systems, including those satisfying Newton's law F=ma. Plot a root locus for each of the compensators just discussed. Does the compensator stabilize the plant?

a. P Feedback

The loop gain is $kG(s) = kK(s)H(s) = k\frac{1}{s^2}$. Using MATLAB the root locus is plotted. The command lines are:

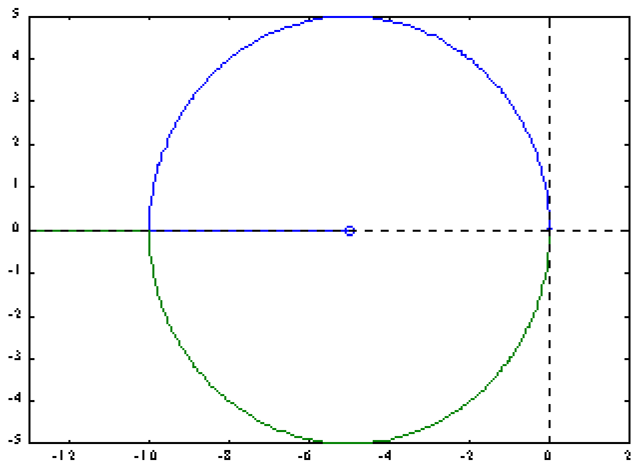
```
num= [1] ;
den= [1  0  0] ;
rlocus(num,den)
```

P feedback does not stabilize this system.



b. PD Feedback

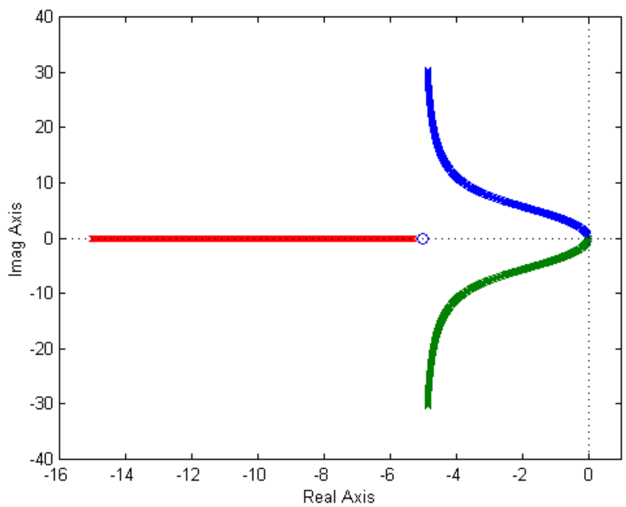
The loop gain is $kG(s) = kK(s)H(s) = k_d \frac{s + {k_p}/{k_d}}{s^2}$. Selecting ${k_p}/{k_d} = 5$ one obtains the RL below. PD feedback does stabilize the double integrator.



c. Practical PD Feedback

In practice, when using PD feedback one must include a filtering pole to smooth out noise in the differentiation process. The loop gain is $kG(s) = kK(s)H(s) = k_d \dfrac{s + {k_p}/{k_d}}{s^2(s+h)}$. Selecting ${k_p}/{k_d} = 5$ and a filtering pole at a large value, here η=15, one obtains the RL below.

Note that the filtering pole changes the RL for high frequencies, but the low frequency behavior is nearly the same as for the pure PD compensator. Thus, practical PD feedback does stabilize the double integrator.

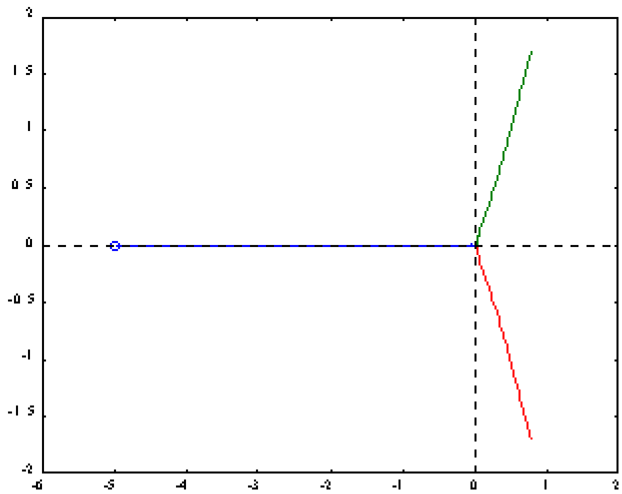


In plotting this RL, MATLAB does not cooperate very well. It is necessary to provide the range of gains k to obtain a good plot. This was accomplished using the command lines:

```
k= [0 : 0.5 : 1000] ;
num= [1  5] :
den= [1  15  0  0] ;
rlocus(num,den,k)
```

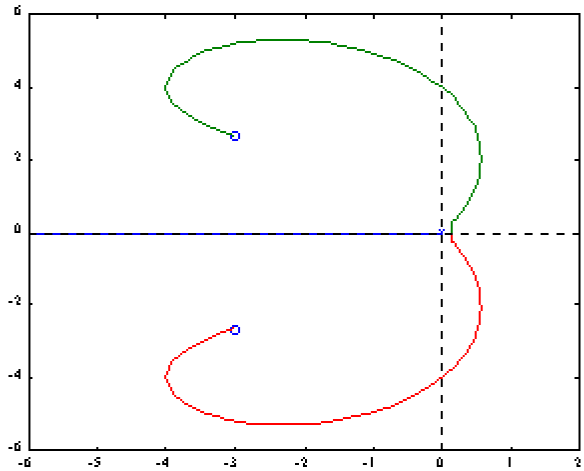
d. PI Feedback

The loop gain with proportional-integral feedback is $kG(s) = k_p \dfrac{s + {k_i}/{k_p}}{s^3}$. The RL appears below for ${k_i}/{k_p} = 5$. PI cannot stabilize the system since it makes the relative degree equal to 3.



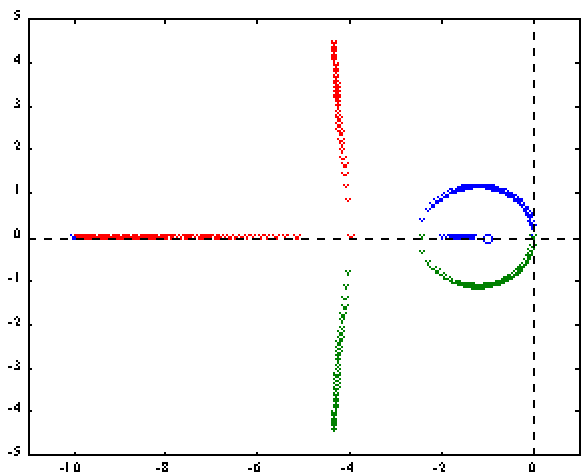
e. PID Feedback

The loop gain with representative PID feedback is $kG(s) = k_d \dfrac{s^2 + 6s + 16}{s^3}$. Note that the PID gains have been selected to obtain a complex conjugate pair of PID zeros. The RL shows that for large enough derivative gains the system is stabilized. The value of the stabilizing gain may be found using the Routh array.



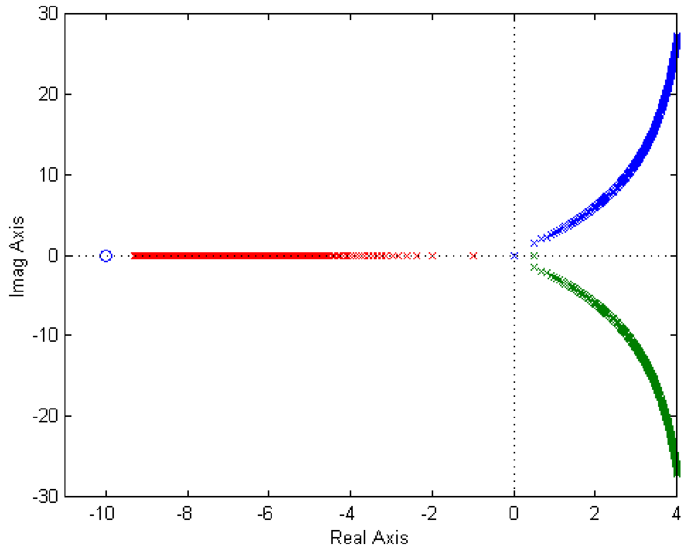
f. Lead Compensator

The loop gain with a representative lead compensator is $kG(s) = k\frac{s+1}{s^2(s+10)}$. The RL shows that the system is stabilized. This is due to the fact that lead compensation here pulls the centroid to the left to the point c= (-10+1)/2= -4.5.



g. Lag Compensator

The loop gain with a representative lag compensator is $kG(s) = k\frac{s+10}{s^2(s+1)}$. The RL shows that the system is not stabilized. This is due to the fact that lead compensation here pushes the centroid to the right to the point c= (10-1)/2= +4.5.



Ziegler-Nichols Tuning of PID Compensator

Three parameters must be adjusted in the PID controller, k_d, k_p, and k_i. Ziegler and Nichols provided a technique for selecting the PID gains that works for a large class of industrial systems.

First, set the derivative and integral gains equal to zero, closing only the P loop. Increase the P gain k_p until the system just oscillates (so that the closed-loop poles are on the jω-axis). Determine the P gain K_m at which this occurs, and the oscillation frequency ω_m at this point. Then, good values of PID gains are computed according to

$$k_p = 0.6K_m, \quad k_d = \frac{k_p p}{4w_m}, \quad k_i = \frac{k_p w_m}{p}.$$

This technique has been found over the years to give very good results for many systems. It relies on the fact that many industrial processes effectively have a relative degree of three due to a dominant low frequency mode, so that as the P gain increases, the closed-loop poles cross the jω-axis and migrate into the right-half plane.

Example- Ziegler-Nichols PID Tuning Using Root Locus

The Ziegler-Nichols technique can be used on actual industrial processes, but also allows design using the root locus.

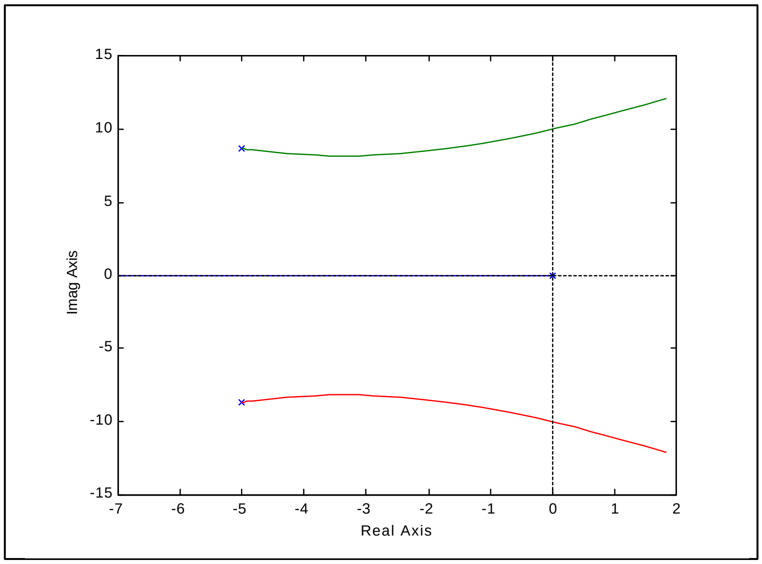
Suppose the plant is given by

$$H(s) = \frac{5}{s(s^2 + 10s + 100)}.$$

Using a P compensator results in the loop gain $k_p H(s)$. The root locus is plotted below.

Using the Routh test (or the MATLAB command 'rlocfind'), one may determine that the gain where the poles cross the jω-axis is equal to K_m=200, and the poles there are at $s = \pm j9.95$, so that ω_m= 9.95 rad/sec.

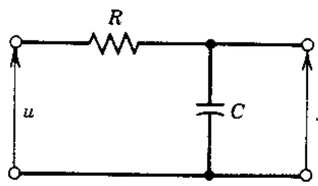
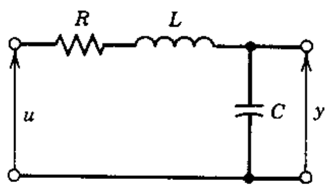
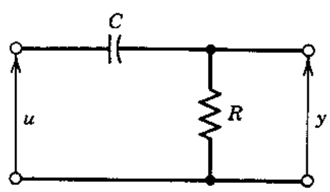
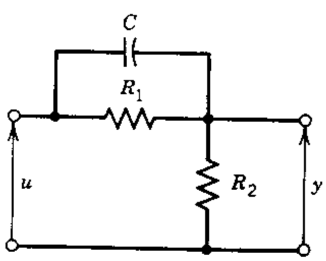
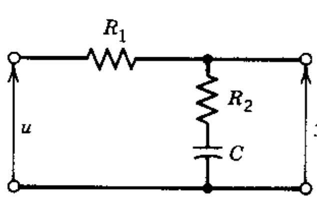
The PID gains are then computed as k_p=120, k_d=9.47, k_i= 380.



Implementing Analog Compensators

Analog compensators may be implemented using electric circuits as shown in the figure. However, today compensators are generally implemented using digital techniques on computers. We shall cover digital compensators later.

TABLE 3.2-1. Network Transfer Functions and State Equations

Network	Transfer Function	State Equations
 Simple lag	$\frac{1}{1+s\tau}$, $\tau = CR$	$\dot{x} = \frac{u-x}{\tau}$ $y = x$
 Quadratic lag	$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ $\omega_n^2 = \frac{1}{LC}$ $\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$	$\dot{x}_1 = x_2$ $\dot{x}_2 = -\omega_n^2 x_1 - 2\zeta\omega_n x_2 + \omega_n^2 u$ $y = x_1$
 Simple lead	$\frac{s\tau}{1+s\tau}$, $\tau = CR$	$\dot{x} = \frac{u-x}{\tau}$ $y = u - x$
 Lead compensator	$\frac{s+z}{s+p}$, $z = 1/\tau$ $p = 1/(\alpha\tau)$ $\alpha = \frac{R_2}{R_1 + R_2}$ $\tau = CR_1$	$\dot{x} = u - px$ $y = u + (z-p)x$
 Lag compensator	$\alpha\left(\frac{s+z}{s+p}\right)$, $z = 1/(\alpha\tau)$ $p = 1/\tau$ $\alpha = \frac{R_2}{R_1 + R_2}$ $\tau = C(R_1 + R_2)$	$\dot{x} = u - px$ $y = \alpha[u + (z-p)x]$