

# EE 4443/4329 – Bode Plots

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## Plotting Transfer Functions

It is very easy to make 3-D plots of transfer function magnitudes and phases using MATLAB. The following example is from Shahian and Hassul, “Control System Design Using MATLAB, Prentice-Hall 1993.

We will demonstrate the process by creating the frequency response surface of a second order system. Consider the system

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Letting  $s = \sigma + j\omega$  we get

$$G(\sigma + j\omega) = \frac{\omega_n^2}{(\sigma^2 - \omega^2 + 2\zeta\sigma\omega_n + \omega_n^2) + j(2\sigma\omega + 2\zeta\omega\omega_n)}$$

For  $\zeta = 0.5$  and  $\omega_n = 1$

$$G(\sigma + j\omega) = \frac{1}{(\sigma^2 - \omega^2 + \sigma + 1) + j(2\sigma\omega + \omega)}$$

This function of two variables will now be plotted for:  $-3 \leq \sigma \leq 3$  and  $-3 \leq \omega \leq 3$ .

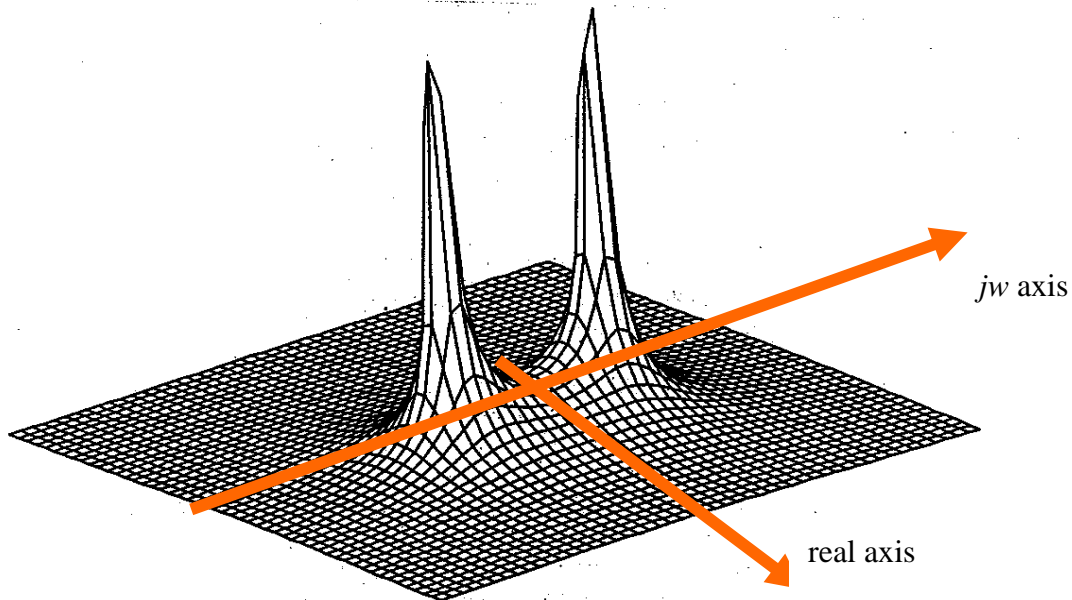


Figure 2-8 Mesh plot of the frequency response surface. An example of 3-D plotting.

```
>> w=linspace(-3,3,50); s=linspace(-3,3,50);
>> [W,S]=meshdom(w,s);
>> re=S.^2-W.^2+S+1; im=2*S.*W+W; den=re+j*im;
>> z=ones(den)./abs(den); mesh(z)
```

Note the use of element-by-element operations for matrices. The plot is shown in Figure 2-8. You can view mesh surfaces from specified viewpoints.

```
>> mesh(z,[azimuth elevate])
```

allows you to specify the azimuth (horizontal rotation) and elevation angles (in degrees). Positive angle for the azimuth rotates the object clockwise, and positive value for the elevation views the object from above (90 degrees is directly overhead). The default viewing angles are [-37.5 30].

Note that more recent versions of MATLAB may require this code to be modified.

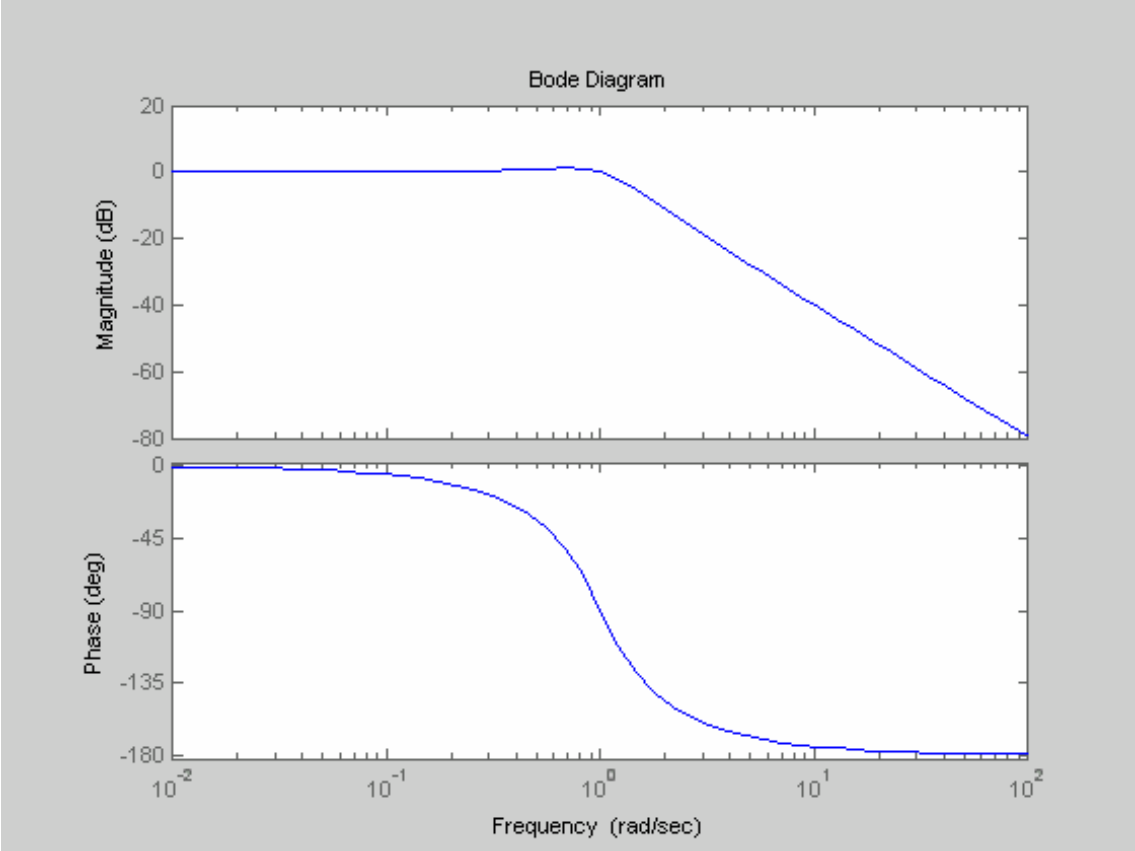
# Bode Plot

Prior to the advent of modern computers and software, making such 3D plots was not easy. In the early 1900s, Bode discovered that it is not necessary to make a full 3D plot of the frequency response to perform control systems design. In fact, he showed that it is only necessary to plot the magnitude and phase along the  $j\omega$  axis. These two plots are known as Bode plots. Based on these two plots, the notions of phase margin and gain margin allow the design of stable feedback systems.

In the figure above, the Bode magnitude plot corresponds to slicing the 3D plot given along the  $j\omega$  axis.

Using MATLAB, the Bode plots for the example given above are easily plotted.

```
num=[1];  
den=[1 1 1];  
bode(num,den);
```



## Rules of Thumb for Making Bode Plots

It is often useful to perform a quick sketch of the Bode plots. This can often be accomplished using some easy Rules of Thumb.

The following rules of thumb work for systems where the separation between poles and zeros is sufficient-- i.e. about 10 times the frequency of the next lowest pole or zero.

### Bode Magnitude Plot Rule of Thumb

The rule of thumb for making the Bode Magnitude Plot is:

*For every pole, the slope of the magnitude decreases by 1.*

*For every zero, the slope of the magnitude increases by 1.*

A slope of 1 is equal to 20 dB/decade or 8dB/octave.

### Bode Phase Plot Rule of Thumb

The rule of thumb for making the Bode Phase Plot is:

*For every pole, the angle of the phase plot decreases by  $90^\circ$ .*

*For every zero, the angle of the phase plot increases by  $90^\circ$ .*

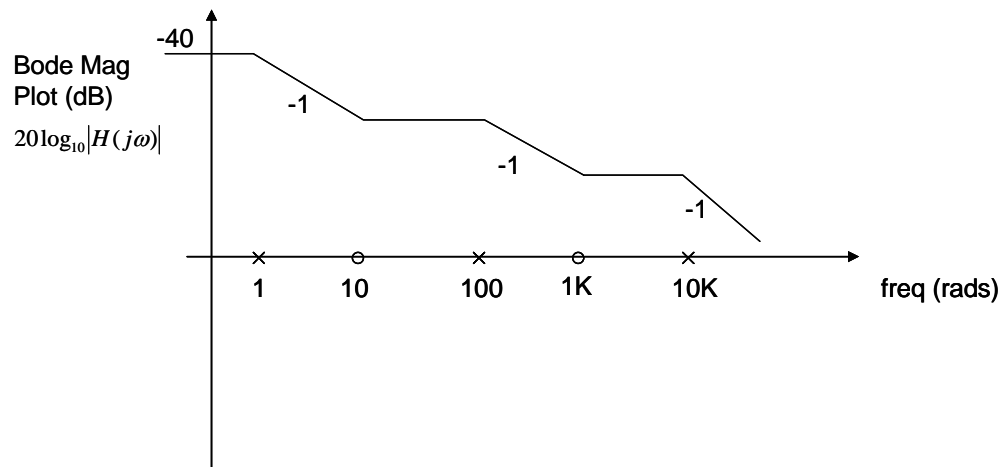
## Procedure for Rough Bode Plots- Example 1

This example shows how to sketch Bode Plots.

The system is  $H(s) = \frac{(s + 10)(s + 1000)}{(s + 1)(s + 100)(s + 10,000)}$

### Magnitude Plot

1. Draw frequency axis and insert poles and zeros on the frequency axis.
2. Draw mag plot using rule of thumb.



3. To calibrate the y axis, set  $s=0$  and find

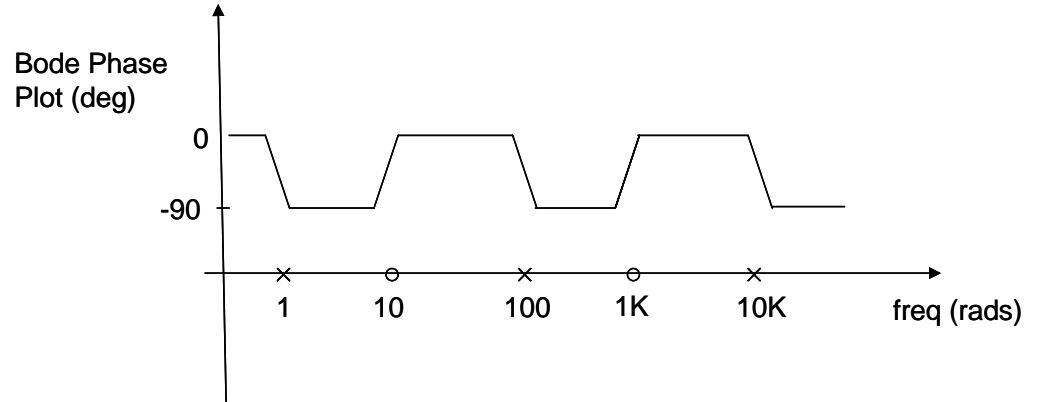
$$H(0) = \frac{(10)(1000)}{(1)(100)(10,000)} = 10^{-2}$$

or in dB

$$20\log_{10}|10^{-2}| = -40 \text{ dB}$$

### Phase Plot

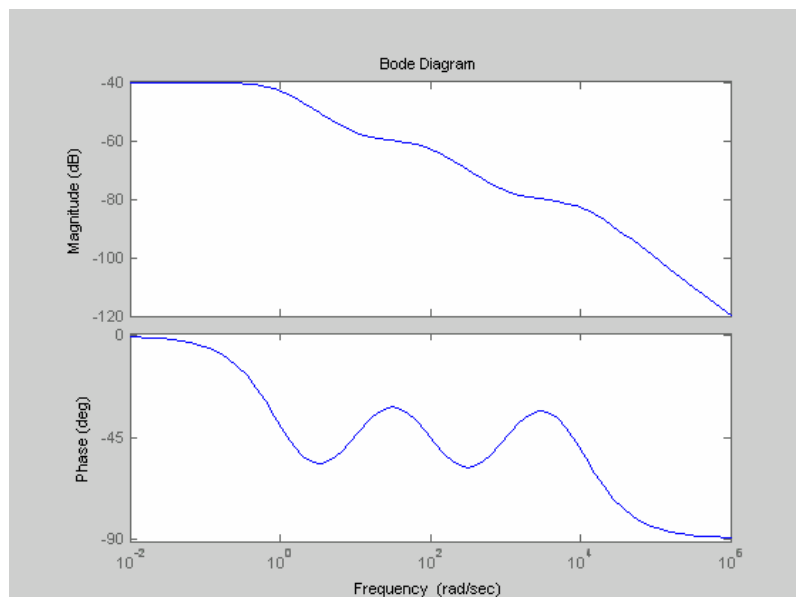
1. Draw frequency axis and insert poles and zeros on the frequency axis.
2. Draw phase plot using rule of thumb.



### MATLAB Accurate Bode Plot

The Bode plot is easy to make with MATLAB.

```
num=conv([1 10],[1 1000])
den=conv([1 1],[1 100])
den=conv(den,[1 10000])
bode(num,den)
```



Note that the convolution function *conv* is used to multiply polynomial factors to get the transfer function numerator and denominator, which turn out to be

$$\begin{aligned} \text{num} = & \\ & 1 \quad 1010 \quad 10000 \\ \text{den} = & \\ & 1 \quad 10101 \quad 1010100 \quad 1000000 \end{aligned}$$

Note that in the phase plot, there is not enough separation between the frequencies of the individual poles and zeros to allow the angle to go fully to -90 deg.

## Some Refinements

There are a few rules for refining your Bode sketches-

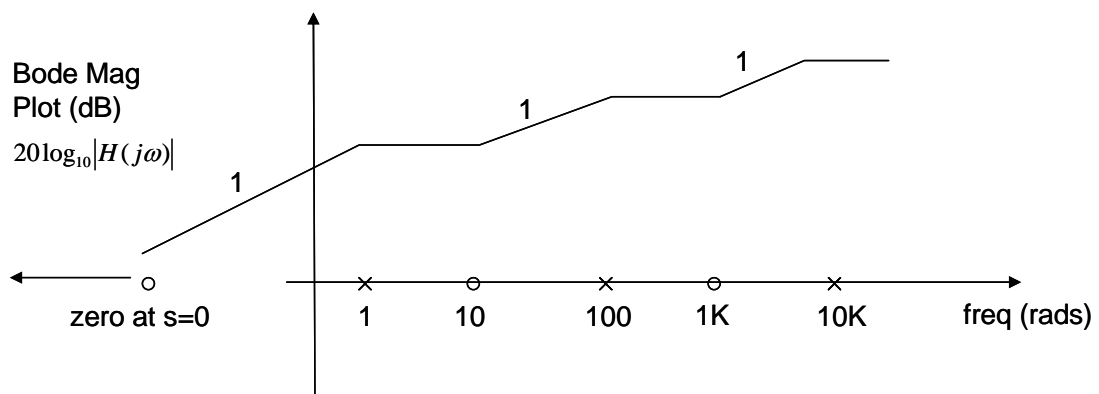
1. The transitions in mag slope and phase angle begin at a frequency about 10 times below the pole or zero frequency, and are complete about 10 times above that frequency.
2. The angle has changed by half its total amount at the pole or zero frequency.
3. For a single pole or zero, the magnitude has changed by 3dB at the pole or zero frequency.

## Example 2- initial mag slope at zero frequency

Sometimes there is an initial slope on the magnitude. Consider the system of Example 1 but with an extra zero-

$$H(s) = \frac{s(s+10)(s+1000)}{(s+1)(s+100)(s+10,000)}$$

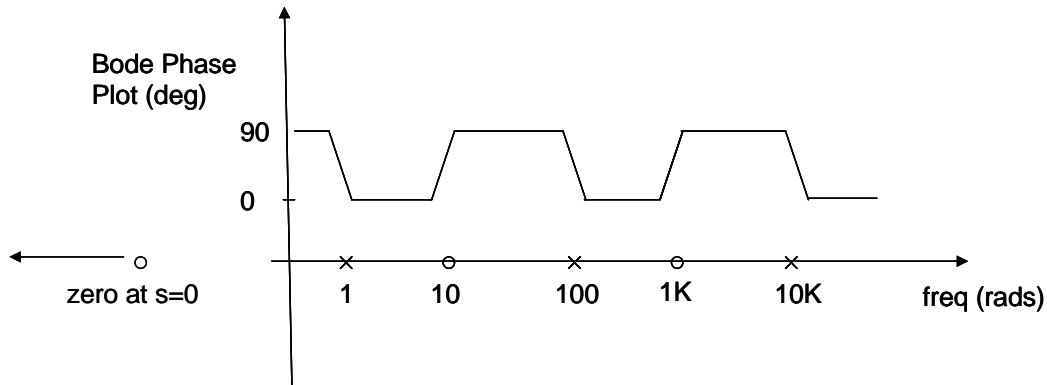
### Magnitude Plot



Note that the zero at  $s=0$  makes the initial slope be plus 1. Note that this magnitude plot is the same as that of example 1, but with all slopes increased by 1.

The zero at  $s=0$  makes this system behave like a differentiator at low frequencies.

### Phase Plot



Note that the zero at  $s=0$  makes the initial angle be 90 deg. Note that this phase plot is the same as that of example 1, but with all angles increased by 90 deg.

### Bode Plot of Complex Pole Pair

Consider the transfer function

$$H(s) = \frac{\omega_n^2}{s^2 + 2\alpha s + \omega_n^2} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \equiv \frac{\omega_n^2}{\Delta(s)}$$

which has a complex pole pair. This yields a resonant peak in the Bode plot.

In fact, the *quality factor*

$$Q = \frac{1}{2\zeta} = \frac{\omega_n}{2\alpha}$$

measures the sharpness of the resonant peak in the Bode plot, as shown in the figure below from Dorf and Bishop. Note that this is effectively determined solely by the damping ratio. The poles are complex if  $Q > 1/2$ .

In terms of the quality factor one may write the characteristic polynomial in the non-dimensional form

$$\Delta(s) = \left(\frac{s}{\omega_n}\right)^2 + \frac{1}{Q}\left(\frac{s}{\omega_n}\right) + 1$$

The resonant frequency is given for  $\zeta \leq 0.7$  by

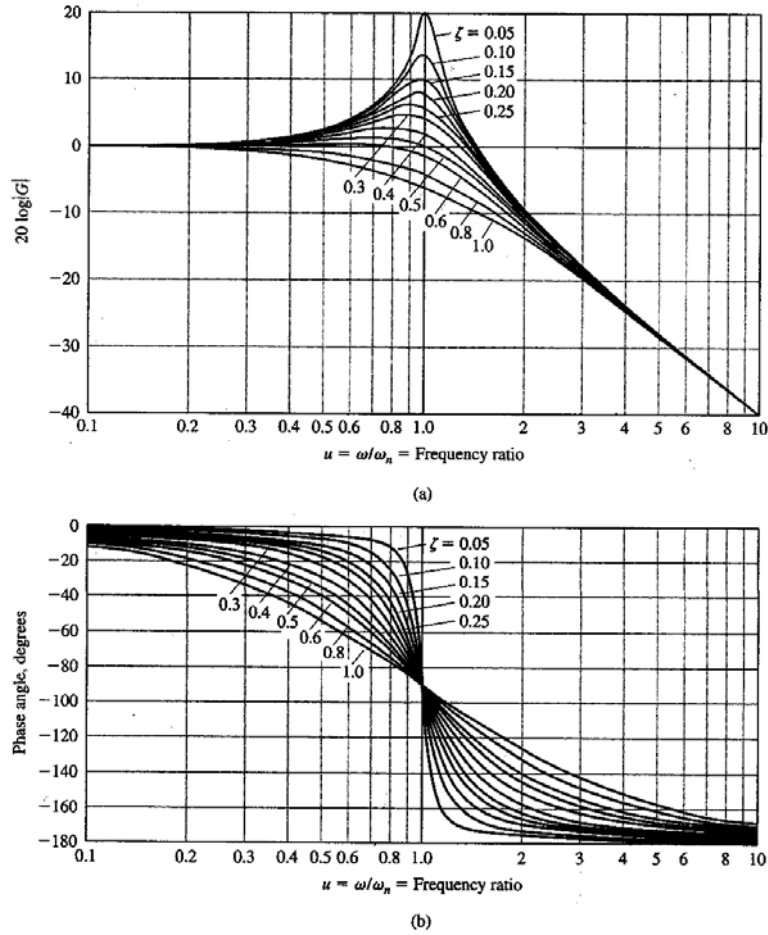
$$\omega_r = \omega_n \sqrt{1 - 2\zeta^2} .$$

The maximum value of the Bode plot at resonance is given by

$$M_{p\omega} = \frac{1}{2\zeta\sqrt{1-\zeta^2}} .$$

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Chapter 8 Frequency Response Methods



**FIGURE 8.10**  
Bode diagram for  $G(j\omega) = [1 + (2\zeta/\omega_n)j\omega + (j\omega/\omega_n)^2]^{-1}$ .

In terms of the rules of thumb, there are two poles at the resonant frequency. This means that the slope in this example changes from 0 to -2. The angle changes from 0 deg to -180 deg.